

# Task: NIM

## Nim with a twist



XXIII OI, Stage I. Source file `nim.*` Available memory: 256/64 MB.

19.10–16.11.2015

The favorite pastime of Alice and Byteasar is playing *Nim*. The game starts with an arbitrary initial arrangement of tokens into heaps. The two players move alternately, where a single move consists in choosing an arbitrary heap and removing from it any positive number of tokens. The player who cannot make a move loses\*.

Alice has just proposed another game of Nim. To make it more interesting, this time they have decided to alter the rules slightly. Namely, Alice has distributed the  $m$  tokens into heaps of sizes  $a_1, a_2, \dots, a_n$ , as usual. Then, before the game commences, Byteasar may remove some of the heaps from play. The number of removed heaps must be divisible by a predetermined number  $d$ , and at least one heap must remain. Afterwards, they are going to play a regular game of Nim, with Alice moving first.

Let  $k$  denote the number of valid subsets of heaps such that Byteasar can have them removed and be able to win the game afterward, regardless of Alice's moves. Your task is to provide the remainder of  $k$  divided by  $10^9 + 7$ .

### Input

The first line of the standard input contains two positive integers  $n$  and  $d$ , separated by a single space, specifying the number of heaps and the divisor of the number of heaps that Byteasar may remove respectively.

The second line describes the heaps: It contains a sequence of  $n$  positive integers  $a_1, a_2, \dots, a_n$ , separated by single spaces, such that  $a_i$  is the number of tokens in the  $i$ -th heap.

### Output

The first and only line of the standard output should contain a single integer: the number (modulo  $10^9 + 7$ ) of different subsets of heaps that Byteasar can remove, ensuring his subsequent victory.

### Example

For the input data:

```
5 2
1 3 4 1 2
```

the correct result is:

```
2
```

**Explanation of the example:** Byteasar can remove either 2 or 4 heaps. He is going to win only if he removes one heap with 1 token and another one with 4 tokens; there are two sets of heaps with this property.

#### Sample Grading Tests:

**1ocen:**  $n = 9$ ,  $d = 2$ , the result is 0,

**2ocen:**  $n = 12$ ,  $d = 4$ ,

**3ocen:**  $n = 30$ ,  $d = 10$ , every heap has 30 tokens,

**4ocen:**  $n = 500\,000$ ,  $d = 2$ , every heap has only 1 token.

### Grading

The set of tests consists of the following subsets. Within each subset, there may be several test groups.

In all the subsets, the conditions  $n \leq 500\,000$ ,  $d \leq 10$ ,  $a_i \leq 1\,000\,000$  are satisfied. Moreover, the total number of tokens  $m = a_1 + a_2 + \dots + a_n$  does not exceed  $10\,000\,000$ . Note that the memory limit varies among subsets.

---

\*More information about the game Nim, including a description of a winning strategy, can be easily found on the Internet.

Subset	Property	Memory Limit	Score
1	$n \leq 20, a_1, \dots, a_n \leq 1000$	256 MB	10
2	$n \leq 10\,000, a_1, \dots, a_n \leq 1000$	256 MB	18
3	$d \leq 2$	256 MB	25
4	none	256 MB	27
5	none	64 MB	20